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ESTIMATING THE STATISTICAL PROPERTIES OF CRACK GROWTH FOR SMALL CRACKS

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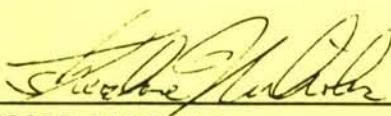
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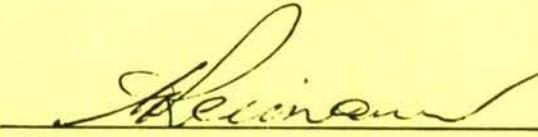
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A series of statistical studies were conducted on two sets of fatigue crack growth data which were generated under variable amplitude loading from cracks that initiated in 7475-T7351 Aluminum down in the bore of typical aircraft quality prepared holes. The study considered fatigue crack growth (life) behavior and fatigue crack growth rate behavior. Generally, the study showed that small crack growth behavior was similar to that observed for larger cracks and that a stress intensity factor parameter could be used to describe the mean trend of the fatigue crack growth rate behavior via a power law relationship. | | |

FOREWORD

This study was conducted by the authors during the time frame from September 1979 to September 1980 under USAF contract F33615-78-C-5184 for the Air Force Wright Aeronautical Laboratories/the Materials Laboratory. Dr. Theodore Nicholas of the Metals and Ceramics Division of the Materials Laboratory was the project monitor for this study of the application of non-linear fracture mechanics (NLFM) parameters to the study of fatigue crack growth. One of the purposes of considering the small crack data presented in this report was to determine if NLFM parameters were required to characterize small crack behavior. For the data sets considered, the data were adequately described by the linear elastic fracture mechanics approach.

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SECTION 1

INTRODUCTION

This report investigates some of the statistical characteristics of fatigue crack growth (FCG) behavior for small cracks. The characteristics covered include FCG and fatigue crack growth rate (FCGR) variability, crack length serial effects, and the relationship of FCGR to stress intensity factor (K_{max}). Crack length serial effects can be described as the relationship between observed FCGR's for adjacent increments. Because the material is expected to be similar in adjacent increments, the FCGR's are also expected to be similar. Besides presenting these descriptive statistics, a comparison is made between large and small cracks.

Various aspects of the statistics of FCG and FCGR have been studied by others. Mukherjee and Burns¹ studied the use of regression models relating FCGR to the stress intensity factor and discussed the errors associated with these methods. The effects of different differentiation methods and increment sizes on FCGR variability for constant stress intensity factor conditions was investigated by Artley, et al². Shaw and LeMay³ recently compared various methods of determining intrinsic material scatter. An extensive set of FCG data was collected for constant amplitude stress conditions by Virkler, et al.⁴ in a study aimed at evaluating various statistical distributions to FCGR at different stress intensity factor levels. As part of their study Virkler, et al. also attempted to correlate the variability in FCGR behavior with the variability in FCG life behavior. The Virkler, et al. FCG data set was also used as the basis for a recent study by Berens, et al.⁵ which modeled FCG lives from FCGR data derived from the original FCG data set. Simulations were used in the Berens et al. study to validate the model that was developed. Some results developed in these earlier studies will be compared to results generated in this study.

The data for the current study were obtained from a study⁶ which was designed to evaluate the influence of the quality of a fastener hole on structural durability of 7475-T7351 Aluminum. While this study investigated a wide range of loading and fastener conditions, only two drilling machines were used: The Quackenbush Model No. QDA-100 and the Winslow Spacematic Model No. S1. Besides being different manufacturers, these represent two different procedures. The Quackenbush process is a two-stage drill and ream procedure while the Winslow process includes only drilling.

The FCG data selected for the current study were taken from no-load-transfer specimens which had properly drilled holes and which were tested under a F-16 fighter load spectrum. The FCG data sets were further censored to include only those specimens exhibiting FCG through the crack length interval from 0.0025 to 0.0375 inches. While the exact shape of each crack is unknown, the investigation revealed that typically the cracks were growing in the bore of the hole in a semielliptical shape.

SECTION 2

VARIABILITY OF LIFE

In the Reference 6 study, FCG data were collected periodically with a constant time increment frequency of 400 flight hours (the length of the repeating load block). Interpolation was used to create a set of FCG data with a constant crack growth increment (Δa) of 0.005 inches through the same interval. A constant Δa facilitates the investigation of the effect of stress intensity factor on FCGR and of serial effects on FCGR. The interpolated data were normalized to zero life at a crack length of 0.0025 inches and since the crack grew to 0.0375 inches this gives seven increments to calculate FCGR's.

The two sets of FCG data are presented in Figures 1 and 2. Figure 1 contains the data obtained from specimens prepared with the Quackenbush tooling, likewise, Figure 2 contains the data for the test series where the hole was prepared using the Winslow tooling. The code names for these two data sets are QPF and WPF. The P stands for proper drilling and the F for fighter spectrum. The Q and W have the obvious meanings of Quackenbush and Winslow.

In the WPF group, as shown by Figure 2, the FCG behavior from two specimens stand out from the rest. These two specimens are coded WPF 7 and WPF 11 in the data report. The values of the test statistics (r ratios) are 0.76 and 0.81 for WPF 7 and WPF 11, respectively; both of these values lie above the 99.5 percentile of the distribution of r ratios. If the fatigue lives for WPF 7 and WPF 11 came from the same distribution as the rest of the WPF specimens fatigue lives, there would be less than one half of one percent chance that values for the r ratio as high as those observed would occur. This indicates that something different happened in the testing of specimens WPF 7 and WPF 11.

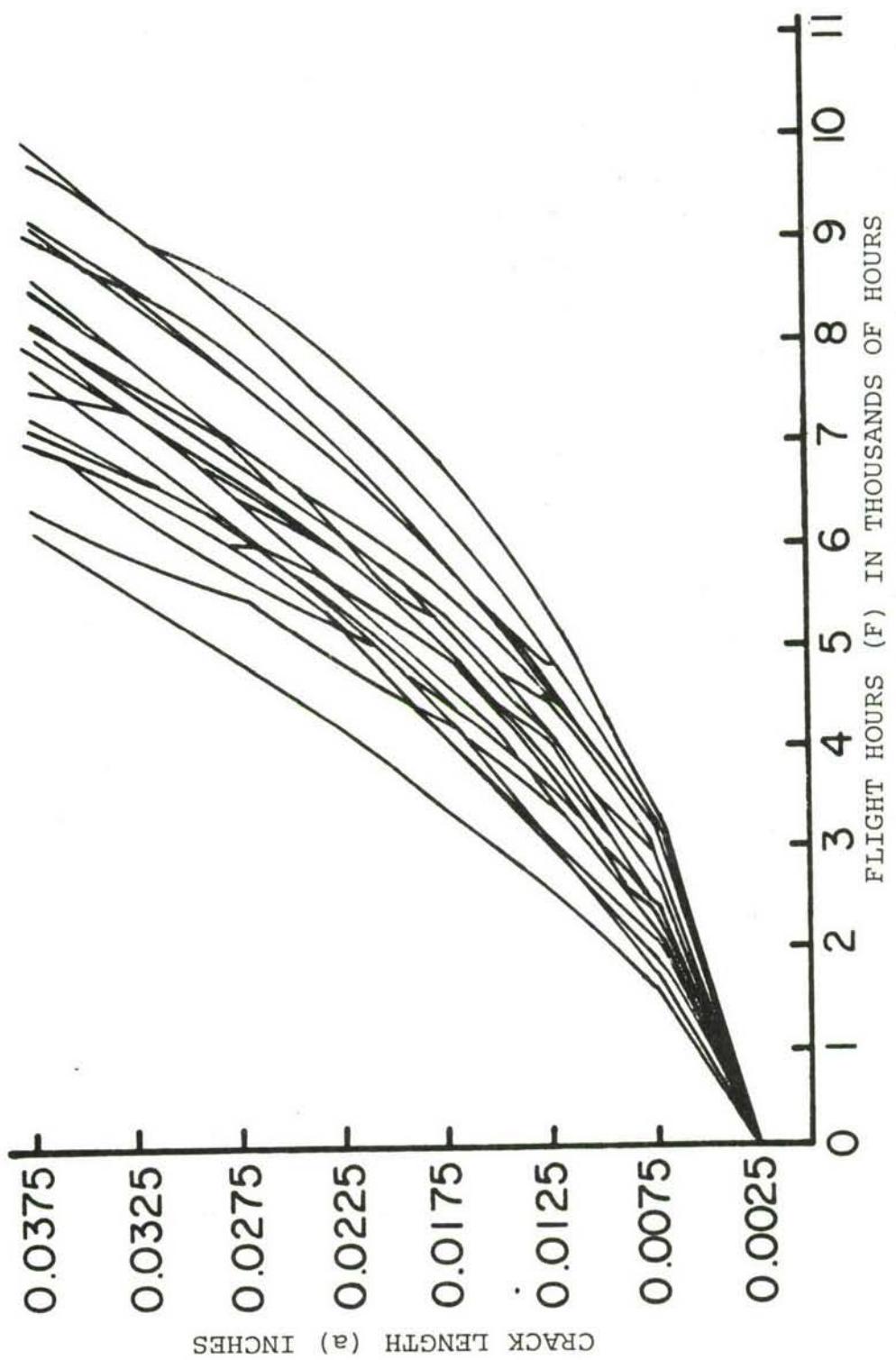


Figure 1. Fatigue Crack Growth Behavior Exhibited by Open Hole Specimens, Prepared with the Quackenbush Equipment and Subjected to an F-16 Fighter Wing Stress History (Referred to as the QPF Data Set).

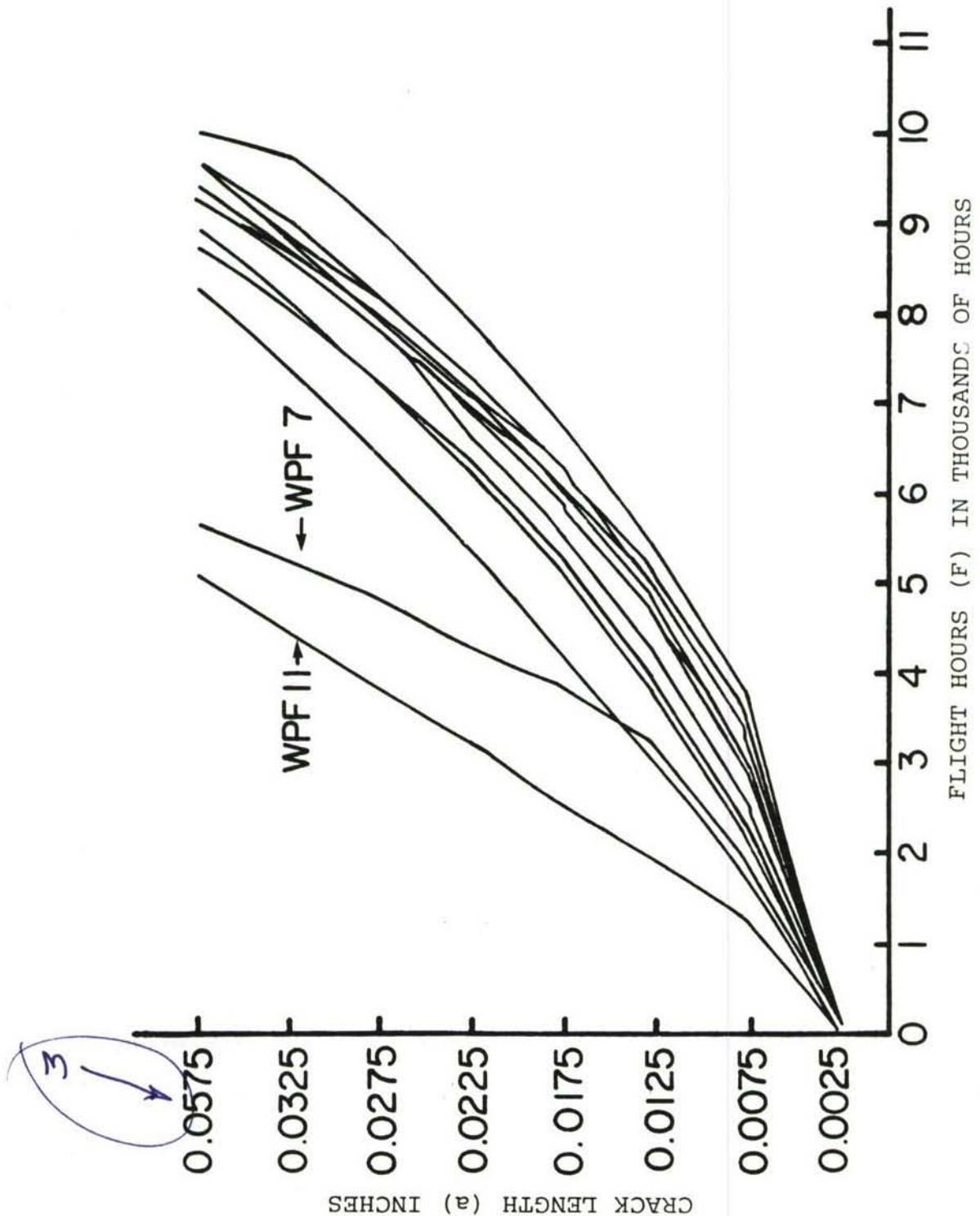


Figure 2. Fatigue Crack Growth Behavior Exhibited by Open Hole Specimens, Prepared with the Winslow Equipment and Subjected to an F-16 Fighter Wing Stress History (Referred to as the WPF Data Set).

Although there is no record of abnormalities in the test conditions for these two specimens, they will be removed from subsequent analyses. The main purpose of this study is to collect background data for the development of a model of FCG lives based on FCGR. Initial modeling efforts require a simplicity which cannot be obtained with outliers present. Henceforth, in this report the term WPF set will refer to the set with WPF 7 and WPF 11 excluded, unless otherwise indicated.

Summary statistics for "lifetimes," in flight hours, for the crack to grow from 0.0025 inches to 0.0375 inches are listed in Table 1. Three sets of statistics are presented: one for the QPF group, one for the WPF group, and one for the WPF group with the two outliers mentioned in the previous paragraph. The mean, standard deviation, and coefficient of variation of both lifetimes and log lifetimes are given in the table. The QPF set seems to grow faster than the WPF set, particularly when the two outliers are ignored.

Typically, a lognormal distribution is used to describe the distribution of lifetimes or of fatigue life ratios. Life ratios are the ratio of predicted life to actual life. For these data, life ratios were calculated using the mean lifetime for the set (QPF or WPF) as the predicted life. Figure 3 contains plots of the observed cumulative distribution functions on a lognormal probability scale. As can be seen in Figure 3, all of the data fall very close to a straight line which is indicative of a lognormal distribution for life ratios. This can be substantiated with the Shapiro Wilk W test. This is a test for the applicability of the normal distribution and when applied to log values can be used to test for the lognormal distribution. The W statistics for the QPF set and the WPF set without the two outliers are 0.9741 and 0.9462, respectively. In this test, low values represent non-normality (or non lognormality). The values of W for the QPF and WPF without outliers sets are between the 50th and 90th percentiles of the W distribution, implying that the assumption of a lognormal distribution is reasonable.

TABLE 1

SUMMARY STATISTICS FOR LIFETIMES FOR CRACK GROWTH FROM
0.0025 INCHES TO 0.0375 INCHES

| Data Set | Mean life (Flt. Hrs.) | Stand. Dev. of Life (Flt. Hrs.) | Coeff. of Var. of Life | Mean Log Life (Log Flt. Hrs.) | Stand. Dev. of Log Life (Log Flt. Hrs.) | Coeff. of Var. of Log Life |
|---------------------|-----------------------|---------------------------------|------------------------|-------------------------------|---|----------------------------|
| QPF | 8001 | 1030 | 0.13 | 3.90 | 0.056 | 0.014 |
| WPF | 9365 | 569 | 0.06 | 3.97 | 0.026 | 0.0066 |
| WPF (with outliers) | 8796 | 1541 | 0.18 | 3.94 | 0.091 | 0.023 |

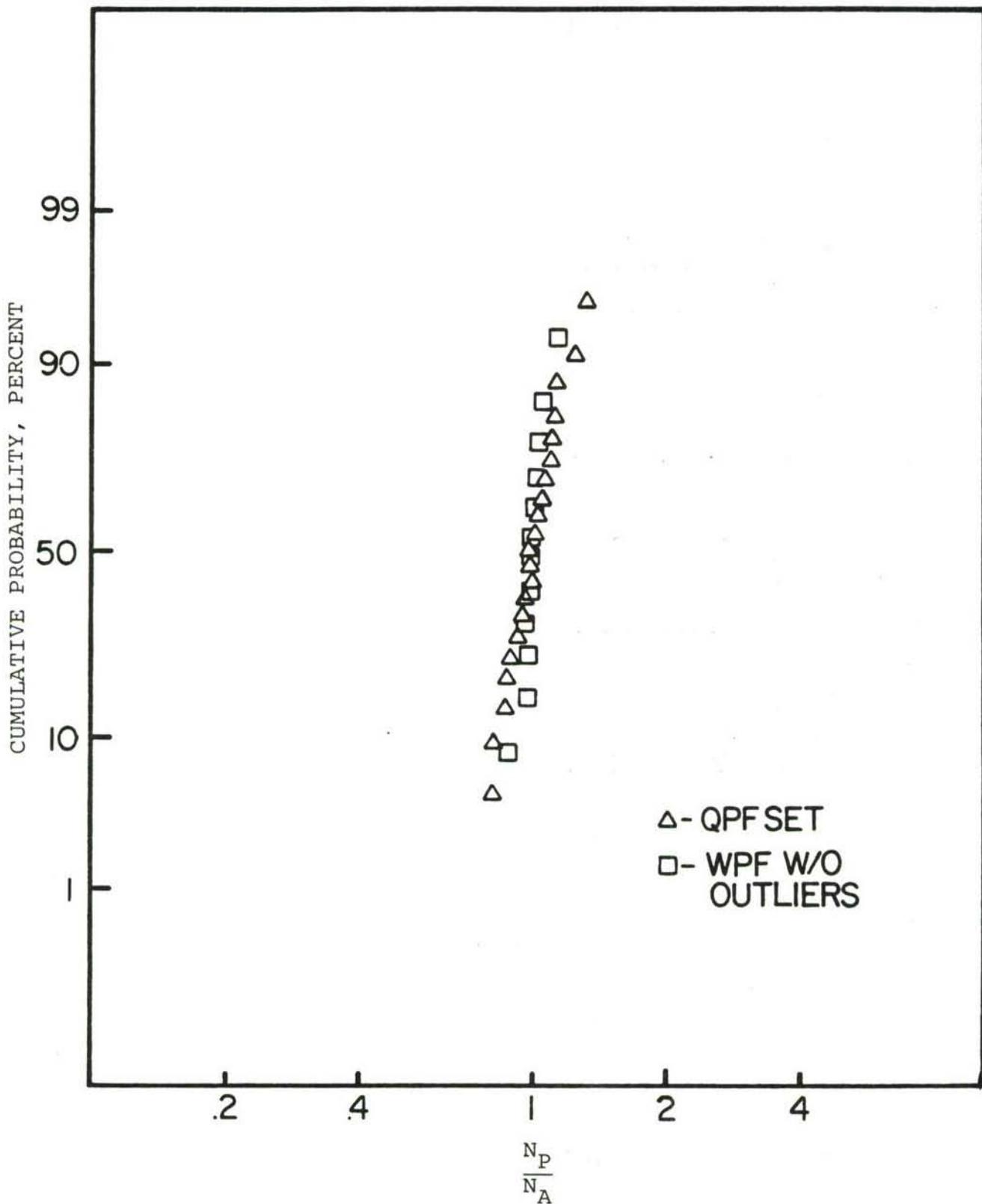


Figure 3. Fatigue Life Ratios Cumulative Distributions.

An F ratio test can be used to test for the equality of standard deviations of log lives. The standard deviation of a sample is the square root of the sample variance. It is well known that the ratio of variances of samples from two different normal populations with the same variance has an F distribution. If the observed F ratio is near one, then it would seem likely that the variances are the same. If it is much larger or smaller than one, the variances are probably different. The F ratio (QPF over WPF) for comparing the QPF set to the WPF set without the outliers was 4.63. We would expect the F ratio to be less than 2.65 about ninety-five percent of the time. Thus, we feel confident that the variances are different.

The mean of log life ratios is always close to zero, simply because of the way they are calculated. Therefore, it is impossible to see any differences in mean log lives in Figure 3. To investigate this possibility, the t test was used. The value of t for testing for a difference in means for the QPF set and the censored WPF set was 4.90. This is well above the 99.5 percentile of a t with 32 d.f. distribution (2.741). In other words, there is less than a one-half of one percent chance of getting a value of 4.9 or larger for t if the two means were the same. Therefore, it is fairly certain the mean log lifetimes are different.

SECTION 3
VARIABILITY IN FCGR

The variability in FCGR for the short crack data analyzed showed similar characteristics to FCGR variability observed in studies involving longer cracks. The mean, standard deviation, and coefficient of variation of FCGR are listed as a function of crack length in Table 2. The FCGR data were created using the secant method on 0.005 inch increments. The statistics are calculated for each crack growth increment within each data set. The crack length (a) given is the midpoint of the increment. The only readily apparent trend is that the mean FCGR increases with the crack length. On closer observation, the coefficients of variation for the QPF specimens are noted to be generally larger than those of the WPF group, and the mean FCGR's for QPF specimens are noted to be generally higher than for WPF specimens in each increment considered.

In both cases, the coefficients of variation are within the bounds seen by other studies. In particular, a study by Artley, et al.² showed coefficients of variation for secant method differentiation procedures ranging from about 35% for a 0.008 inch increment to 15% for a 0.034 inch increment as shown by Figure 4. Figure 4 also provides a comparison between several studies including the Artley, et al. study, a study by Virkler, et al.⁴, the fastener hole quality study, and a study by Shaw and LeMay³. A trend in coefficient of variation as a function of increment size is also illustrated in this figure. The materials employed in these studies were 7075-T6 Aluminum (Artley, et al.²), 2024-T3 Aluminum (Virkler, et al.⁴), and 4140 Steel (Shaw and LeMay³). It is believed that the accuracy of the crack length measurements is likely to be the cause for the differences seen. The higher measurement accuracy associated with electron fractography would explain the fact that the coefficient of variation in the fastener hole quality study is lower than that observed by Artley, et al.², who employed light microscopes (10-30x).

TABLE 2

MEAN, STANDARD DEVIATION, AND COEFFICIENT OF VARIATION OF FCGR AS A FUNCTION OF CRACK LENGTH

| Crack Length | K_{max} (ksi/inches) | QPF SET/21 SPECIMENS | | | WPF SET/12 SPECIMENS + | |
|-------------------------------------|---------------------------|----------------------|--------------------------------|--------------------------------|------------------------|--------------------------------|
| | | Mean FCGR | Standard Deviation FCGR* | Coefficient of Variation | Mean FCGR* | Standard Deviation FCGR* |
| 0.005 | 13.09 | 2.07 | 0.47 | 0.23 | 1.80 | 0.48 |
| 0.010 | 17.08 | 3.70 | 0.66 | 0.29 | 3.14 | 0.24 |
| 0.015 | 19.49 | 4.75 | 0.84 | 0.18 | 4.03 | 0.32 |
| 0.020 | 21.12 | 5.67 | 1.02 | 0.34 | 4.73 | 0.37 |
| 0.025 | 22.31 | 6.67 | 1.13 | 0.17 | 4.37 | 0.60 |
| 0.030 | 23.21 | 7.49 | 1.95 | 0.26 | 6.08 | 0.70 |
| 0.035 | 23.91 | 8.35 | 2.49 | 0.28 | 6.78 | 0.72 |
| AVERAGE COEFFICIENT OF VARIATION | -- | -- | -- | 0.25 | -- | 0.12 |

* THE MEAN FCGR AND STANDARD DEVIATION OF FCGR ARE RECORDED IN 10^{-6} INCHES PER FLIGHT HOUR.

+ OUTLIERS EXCLUDED

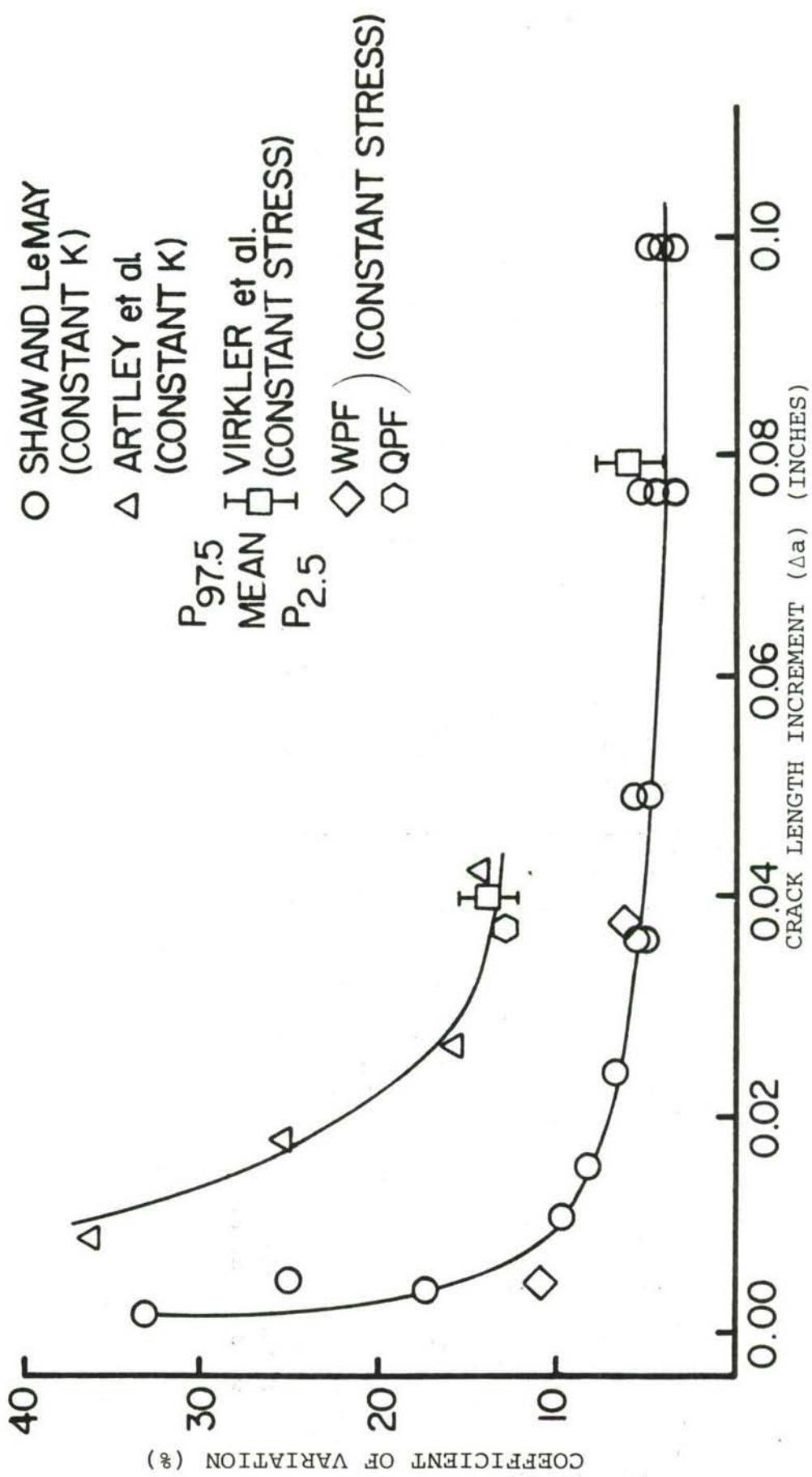


Figure 4. Coefficient of Variation of FCGR Calculated by the Secant Method as a Function of Crack Growth Increment.

SECTION 4

SERIAL CORRELATION

When observations are taken close together either spacially or in time, there is the possibility that they are related. This is referred to as serial correlation or autocorrelation. In this study, observations of FCGR are taken in successive increments of crack growth, therefore serial correlation is suspected.

Cross correlations are used to investigate serial correlations in this study. Cross correlation is a term used to refer to the correlation coefficients of all possible pairs of variables from a specified group of variables. In this case, the seven variables are the FCGR values from the seven increments. Cross correlations are presented in matrix form with the row and column of an entry designating the two variables for which it is a measure of association. Stated differently, the correlation coefficient for the second and fourth variables is entered in the fourth column of the second row. If the full matrix were presented, this correlation would also be entered in the second column of the fourth row. However, to avoid this duplication, usually only the upper right-hand corner of the matrix of correlation coefficients is presented. Tables 3 and 4 contain the correlation matrices for the QPF and WPF without outliers data sets, respectively.

Values of the correlation coefficient range between -1 and 1. A value of 1 or -1 is considered perfect correlation and indicates that one variable can be predicted exactly from the other. That is, if the FCGR is one unit above the mean in the first increment, the FCGR for that specimen in the second increment will also be 1 unit above the mean. A value of 0 for the correlation coefficient indicates no relationship between the two variables; and therefore, the FCGR's in the two increments are independent. Values between 0 and 1 or 0 and -1, indicate varying degrees of the strength of the relationship between the two variables.

TABLE 3
CROSS CORRELATIONS FOR THE QPF DATA

| Crack length (inches) | Crack length (inches) | | | | | | |
|-----------------------------|-----------------------|-------|-------|--------|-------|--------|-------|
| | 0.005 | 0.010 | 0.015 | 0.020 | 0.025 | 0.030 | 0.035 |
| 0.005 | 1 | 0.64 | 0.25 | -0.048 | 0.044 | -0.099 | -0.20 |
| 0.010 | | 1 | 0.80 | 0.42 | 0.59 | 0.25 | 0.33 |
| 0.015 | | | 1 | 0.70 | 0.75 | 0.36 | 0.68 |
| 0.020 | | | | 1 | 0.62 | 0.53 | 0.76 |
| 0.025 | | | | | 1 | 0.40 | 0.56 |
| 0.030 | | | | | | 1 | 0.13 |
| 0.035 | | | | | | | 1 |

TABLE 4

CROSS CORRELATIONS FOR THE WPF DATA WITHOUT OUTLIERS

| Crack length (inches) | Crack length (inches) | | | | | | |
|-----------------------------|-----------------------|-------|-------|-------|--------|-------|-------|
| | 0.005 | 0.010 | 0.015 | 0.020 | 0.025 | 0.030 | 0.035 |
| 0.005 | 1 | 0.56 | -0.15 | -0.52 | -0.46 | -0.68 | -0.61 |
| 0.010 | | 1 | 0.33 | -0.14 | -0.048 | -0.34 | -0.26 |
| 0.015 | | | 1 | 0.60 | 0.47 | 0.26 | 0.51 |
| 0.020 | | | | 1 | 0.52 | 0.75 | 0.80 |
| 0.025 | | | | | 1 | 0.74 | 0.63 |
| 0.030 | | | | | | 1 | 0.79 |
| 0.035 | | | | | | | 1 |

Usually when the variables are ordered, as these are, the correlation coefficient is assumed to be a function of the distance between the two variables. For example, it would be assumed that the first and second, the second and third, the third and fourth, . . . , sixth and seventh variables all have the same correlation coefficient since they are all 1 step apart. This relative separation is referred to as the lag so that the connected pairs represent a lag of 1. For a lag of 2, the pairings would be: first and third, second and fourth, third and fifth, fourth and sixth, and fifth and seventh.

In Tables 3 and 4 all the lag 1 pairings are in the second diagonal of the matrix. The first diagonal contains all 1's since it represents the correlation of each variable with itself. Similarly, the lag 2, 3, 4, 5, and 6 pairings are in the third, fourth, fifth, sixth, and seventh diagonals. (The seventh "diagonal" is the single corner element since the only lag 6 pairing is the first and seventh.)

There are, however, some discrepancies from the assumption that the correlation coefficient is a function of the lag only. The QPF set and the WPF set without the outliers display some inconsistencies. The lag 1 and the lag 3 diagonals have somewhat broader ranges, in the QPF set, than would be expected if the correlation were the same down these diagonals. Similarly, the WPF set without the outliers has a bigger range than expected in the lag 3 diagonal. Also, this set shows an odd pattern of negative correlations in the first two rows and positive correlations elsewhere. This is an indication that something different is happening to fatigue crack growth rates in the first two increments studied here.

More specific conclusions about serial correlation are difficult because of the small sample sizes and the limited number of increments involved. However, from the broad range of correlations seen in Tables 3 and 4 it is clear that some sort of serial correlation exists. Before fatigue crack lifetimes can be adequately modeled from FCGR, this issue must be resolved. Virkler et al.⁴ attempted to predict fatigue crack lifetimes assuming independence of FCGR in adjacent increments. Their technique involved fitting a different, independent lognormal distribution to the FCGR's in each increment. They used these distributions to generate a set of fatigue crack growth rates, which were numerically integrated to get a FCG life. Their simulations predicted the mean FCG life very well but were off considerably in the standard deviation of FCG lives. The predicted standard deviation was approximately one third the actual standard deviation for FCG lifetimes to 31mm.

SECTION 5

THE RELATIONSHIP OF FCGR TO STRESS INTENSITY FACTOR

A Paris power law formulation was used to evaluate the effect of stress intensity factor on FCGR. The form of the power law used is:

$$\frac{da}{dF} = C(K_{max})^m \quad (1)$$

where C and m are constants. The stress intensity factor was estimated using the radial-thru-the-thickness crack formulas suggested by Grandt⁷:

$$K = \sigma\sqrt{\pi a} \left[\frac{0.8733}{0.3245 + (\frac{a}{R})} + 0.6762 \right] \quad (2)$$

where R is the radius of the hole, and a is the crack depth measured from the edge of the hole. No attempt was made to account for the fact that the crack geometry was semielliptical growing away from the center of the hole and down the bore. It is felt that the Equation 2 formula would differ from the actual crack's stress intensity factor by a multiplicative constant although this constant might be a function of crack length if the crack shape changes as the crack grows. The value of the constant is expected to range between 0.7 and 1.0. The stress intensity factor given in Equation 1 is determined by multiplying the maximum stress in the fighter stress history by the stress intensity factor coefficient obtained from Equation 2, i.e.

$$K_{max} = \sigma_{max} \cdot \left(\frac{K}{\sigma}\right) = (34 \text{ ksi}) \cdot \left(\frac{K}{\sigma}\right) \quad (3)$$

Procedures similar to the above have been successfully utilized previously to describe variable amplitude crack growth behavior⁸⁻⁹⁻¹⁰.

By converting the power law into a linear equation through a log transformation, the validity of the power law model can be tested with linear regression procedures. Linear least squares procedures were used to determine the power law constants

presented in Table 5 for both QPF and WPF data sets. A comparison between the individual FCGR data points and the power law determined to describe them is presented in Figures 5 and 6 for the QPF and WPF data sets, respectively.

The parameters in Table 5 indicate that the power law is a good fit to the data. The high correlation coefficients (r) of 0.92 for the QPF set and 0.96 for the WPF set without outliers indicate a strong relationship between FCGR and stress intensity factor.

The F test is used to test the fit of the model and is based on the ratio of two independent estimates of the variability of the FCGR data. One of these estimates measures the variability of FCGR after the model has been fit and the other estimate assumes no model. If the F ratio is near 1, almost no FCGR variability is explained by the model. If the F value is larger than 1, the model has explained some of the variability of FCGR. The F ratio has an F distribution whenever the model does not explain any of the FCGR variability. The probability of getting F values as large as those in Table 5 (P value) would be less than one hundredth of one percent if the model did not explain some of the FCGR variability. Therefore, the power law does explain some of the FCGR variability, or in other words is a good fit.

In the previous section, it was noted that the QPF specimen results showed a faster FCGR than the WPF specimens. The differences in FCGR could be due to differences in crack shape that would lead to different stress intensity factors. Table 6 contains the thru-the-thickness stress intensity factors associated with crack growth rates of 1.6×10^{-6} , 4×10^{-6} , and 10^{-5} inches per flight hour for the QPF data. The stress intensity factors for the corresponding set of FCGR values were obtained for the WPF data set (without outliers) and these are also presented in Tables 6 along with the percent difference in stress intensity factor values. The average percent difference for the WPF set is about 8 percent with a range from 6.4 to 9.5

Table 5. Results of Regression Analysis

| Sample | $\log_{10} \hat{C}$ | \hat{m} | r | F | P value |
|------------------|---------------------|-----------|------|--------|---------|
| QPF | -8.21 | 2.25 | 0.92 | 749.08 | <0.0001 |
| WPF w/o outliers | -8.18 | 2.17 | 0.96 | 940.31 | <0.0001 |

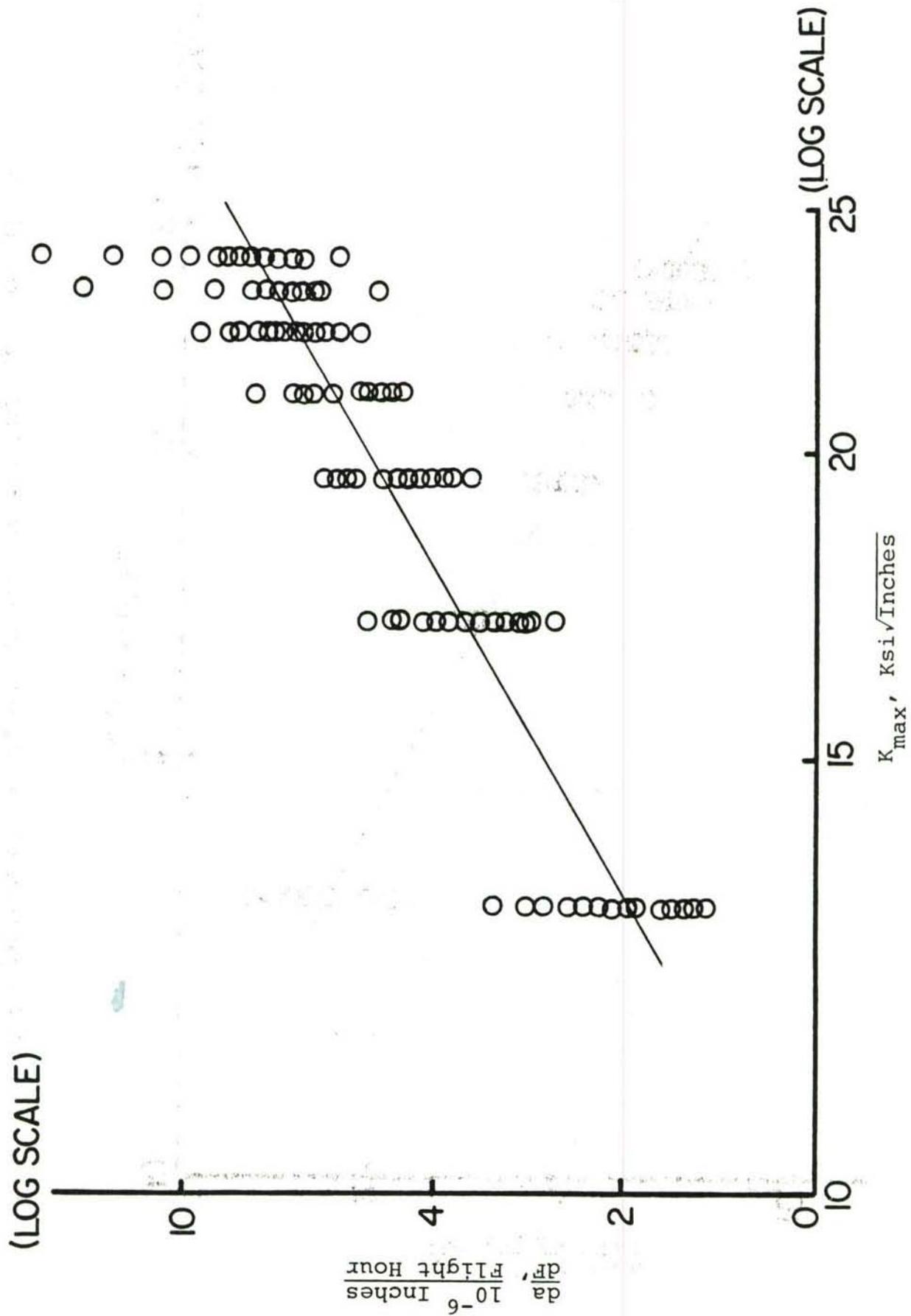


Figure 5. A Comparison of the Power Law Fit and the FCGR Data for the QPF Set.

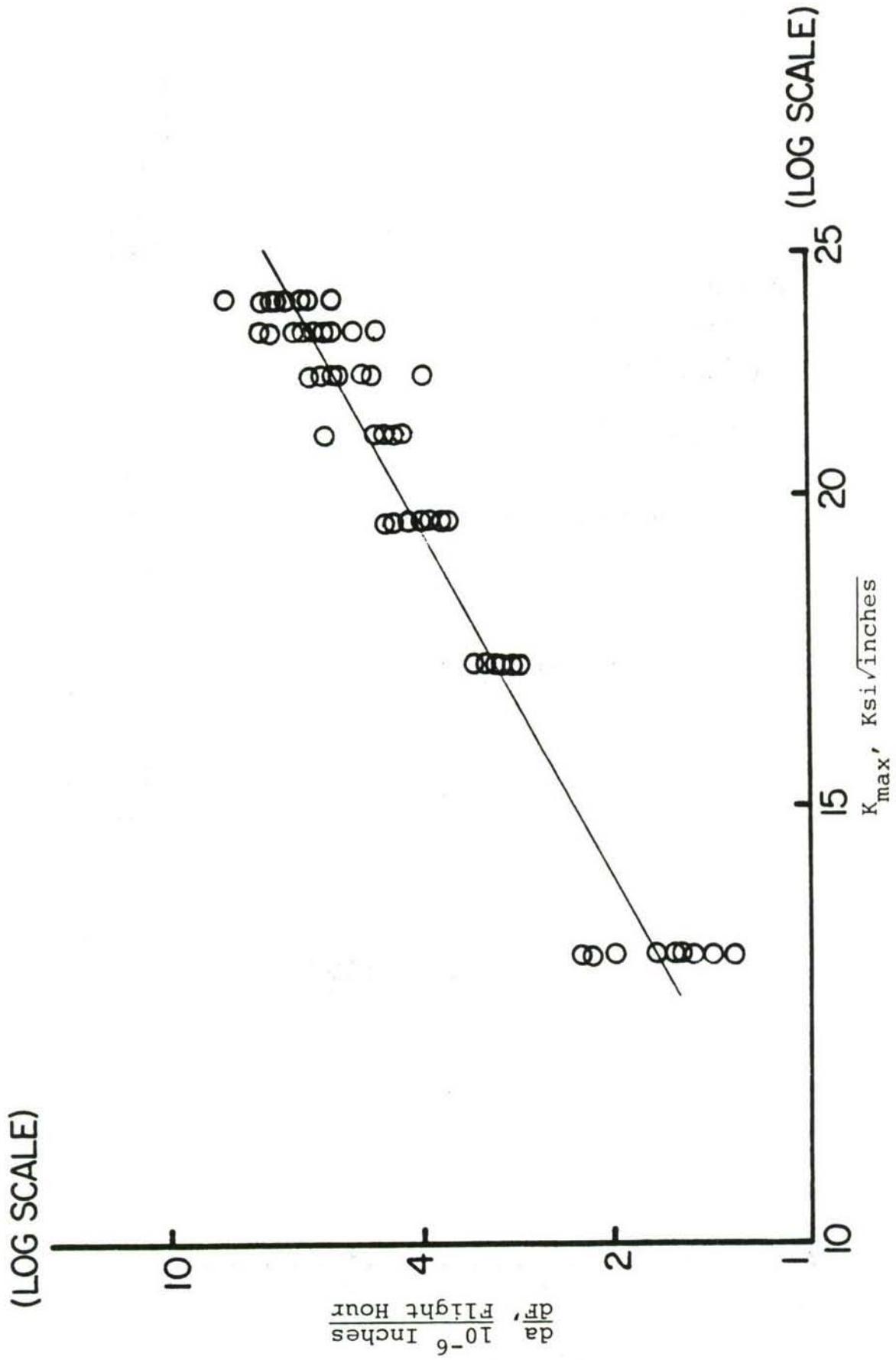


Figure 6. A Comparison of the Power Law Fit and the FCGR Data for the WPF Set Without Outliers.

percent. The difference in stress intensity factors is not surprising since there could be as much as a thirty percent difference in stress intensity factor due to extremes in crack shape. What is surprising, however, is the observation that the differences became greater as the crack grew larger. As cracks get larger, they grow away from the various types of initiation scratch patterns and should grow with a shape that is independent of their starting condition. Thus, what we really expected to observe was the opposite of the trend shown in Table 6.

Table 6. Relative Difference in K_{max} as a Function
of FCGR.

| da/d_F (inches per flight hr.) | K_{max} for the QPF Set $ksiv_{in}$ | % Difference Using QPF as the Base |
|-------------------------------------|--|---------------------------------------|
| 1.6×10^{-6} | 11.87 | 6.4 |
| 4×10^{-6} | 17.84 | 7.9 |
| 1×10^{-5} | 26.83 | 9.5 |

SECTION 6

CONCLUSIONS

The fatigue crack growth (FCG) behavior of the two small crack data sets studied herein is similar to the behavior of large cracks. The fatigue life distributions for these small crack data sets exhibit a lognormal distribution. The coefficients of variation of fatigue crack growth rate (FCGR) for these small crack data sets are within the bounds seen in other studies dealing with longer cracks. The FCGR behavior was shown to be related to the stress intensity factor using a power law. All of the above defined traits have been observed in large cracks.

The two sets of data studied were slightly different. The fatigue cracks in the drilled and reamed holes set grew at a slightly faster rate than the fatigue cracks in the Winslow drilled hole set. This could have resulted from differences in crack shape. Using the power law in reverse showed that the percentage difference in stress intensity factors that would give rise to the same FCGR in both data sets ranged between 6 and 10 percent. Differences in crack shape could give rise to this much error in the calculations of stress intensity factors.

Before an adequate model of FCG lives based on FCGR can be developed, the serial dependencies of FCGR must be accounted for. Serial correlation was observed in both sets of data analyzed here; but, there were some unusual features exhibited for cracks below 0.0125 inch. Due to the limited number of data points specific conclusions about the nature of this correlation cannot be drawn.

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